Int. J. Solids Structures Vol. 22, No. 7, pp. 729–735, 1986
Printed in Great Britain.

0020–7683/86 $3.00+.00
Pergamon Journals Ltd.

SOME PATH INDEPENDENT INTEGRALS FOR MICROPOLAR MEDIA

TIAN-MIN DAI
Liaoning University, Shenyang, People's Republic of China

(*Received* 20 *June* 1985)

Abstract—In this paper by extending the results of Rice, Ouyang, Lu and Chen and using the micropolar field theory proposed by Eringen some path independent integrals for micropolar media are presented.

1. INTRODUCTION

In 1968 Prof. Rice[1] proposed the famous J-integral in two-dimensional fracture mechanics. Since then a great deal of scientific papers have been published all over the world.

In 1982 and 1983 Ouyang and Lu[2, 3] presented some path independent integrals in nonlinear fracture dynamics for two- and three-dimensional cases, respectively. They considered the nonlinear elastic case and the elastic–plastic case. The physical meanings of those path independent integrals and a criterion in nonlinear fracture mechanics are given.

In 1984 Chen[4] derived the three-dimensional J-integral by means of the general potential energy principle and Green's theorem and gave the J-integral a power type. This result may be seen as a fracture criterion.

In 1974 Gross[5] had already investigated the fracture problems for micropolar media in his habilitations thesis.

In 1981 we obtained some new conservation laws for linear micropolar elastodynamics [6]. Then Jin[7] derived in 1983 the theorem on the conservation of the non-conservative field based on the differential variational principle and the conservation law for a certain class of continuum mechanics.

Using the micropolar field theory[8] and expanding the results of Ouyang, Lu and Chen we present some path independent integrals for micropolar media in this paper.

2. CASES IN MICROPOLAR FRACTURE DYNAMICS

2.1. *Path independent integrals in micropolar elastic media*

For the sake of simplicity, we shall use the rectangular coordinates throughout this paper. We assume that there is a plane crack in a three-dimensional micropolar body.

For the dynamical case, the equation of continuity and equations of motion for the micropolar elastic media are as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho v_i) = 0, \tag{1}$$

$$\frac{\partial t_{ji}}{\partial x_j} + \rho(F_i - \dot{v}_i) = 0, \tag{2}$$

$$\frac{\partial m_{ji}}{\partial x_j} + \varepsilon_{ijk} t_{jk} + \rho(M_i - \dot{\sigma}_i) = 0, \tag{3}$$

where ρ, t, x_i, u_i, φ_i, t_{ij}, m_{ij}, F_i, M_i, v_i, σ_i and ε_{ijk} denote the density, the time, the rectangular Cartesian coordinates, the displacement vector, the microrotation vector, the stress tensor, the couple stress tensor, the body force density, the body couple density, the velocity, the spin density and the alternating tensor, respectively.

Here we are concerned with the nonlinear elastic micropolar body with infinitesimal deformations and microrotations. From the micropolar field theory we know that $v_i = \partial u_i/\partial t = \dot{u}_i$, $\sigma_i = j_{ij}\partial\varphi_j/\partial t = j_{ij}\dot{\varphi}_j$ (j_{ij} is the microinertia tensor), the kinetic energy density K and the deformation energy density W are

$$K = \tfrac{1}{2}\rho(v_i v_i + j_{ij}\dot{\varphi}_i\dot{\varphi}_j) \tag{4}$$

$$W = \int (t_{ij}\,\mathrm{d}\varepsilon_{ij} + m_{ij}\,\mathrm{d}\gamma_{ij}), \tag{5}$$

where ε_{ij} and γ_{ij} are the strain tensor and the wryness tensor:

$$\varepsilon_{ij} = \frac{\partial u_j}{\partial x_i} - \varepsilon_{ijl}\varphi_l \tag{6}$$

$$\gamma_{ij} = \frac{\partial \varphi_i}{\partial x_j}. \tag{7}$$

Let S be a surface around some part of the crack border, we may establish the following theorems.

Theorem 1: *The vector integral*

$$\mathbf{D}_1 = \int_{t_0}^{t_1}\int_S \{[W - K - \rho(F_i u_i + M_i\varphi_i)]\mathbf{n} - (t_i\tilde{\nabla}u_i + m_i\tilde{\nabla}\varphi_i)\}\,\mathrm{d}S\,\mathrm{d}t + \int_V \rho(v_i\tilde{\nabla}u_i + \sigma_i\tilde{\nabla}\varphi_i)\,\mathrm{d}V\Big|_{t_0}^{t_1} \tag{8}$$

is path independent for any S around some part of the crack border and any $t_1 > t_0 \geqslant 0$. Here $\mathbf{n}$ is the projection of outer normal vector $\mathbf{v}$ of S on the crack plane, t_i and m_i are the traction vector and the couple vector on S, V is a region bounded by S and crack surfaces and $\tilde{\nabla} = \mathbf{i}(\partial/\partial x) - \mathbf{j}(\partial/\partial y)$.

Proof: Considering a closed surface S, not including the crack border inside, we have

$$\begin{aligned}
\int_{t_0}^{t_1}\int_S (t_i\tilde{\nabla}u_i + m_i\tilde{\nabla}\varphi_i)\,\mathrm{d}S\,\mathrm{d}t &= \int_{t_0}^{t_1}\int_S (t_{ji}v_j\tilde{\nabla}u_i + m_{ji}v_j\tilde{\nabla}\varphi_i)\,\mathrm{d}S\,\mathrm{d}t \\
&= \int_{t_0}^{t_1}\int_V \left[\frac{\partial}{\partial x_j}(t_{ji}\tilde{\nabla}u_i) + \frac{\partial}{\partial x_j}(m_{ji}\tilde{\nabla}\varphi_i)\right]\mathrm{d}V\,\mathrm{d}t \\
&= \int_{t_0}^{t_1}\int_V \left[\frac{\partial t_{ji}}{\partial x_j}\tilde{\nabla}u_i + t_{ji}\tilde{\nabla}\left(\frac{\partial u_i}{\partial x_j}\right) + \frac{\partial m_{ji}}{\partial x_j}\tilde{\nabla}\varphi_i + m_{ji}\tilde{\nabla}\left(\frac{\partial \varphi_i}{\partial x_j}\right)\right]\mathrm{d}V\,\mathrm{d}t \\
&= \int_{t_0}^{t_1}\int_V [\rho(\dot{v}_i - F_i)\tilde{\nabla}u_i + t_{ji}\tilde{\nabla}(\varepsilon_{ji} + \varepsilon_{jil}\varphi_l) + \rho(\dot{\sigma}_i - M_i)\tilde{\nabla}\varphi_i \\
&\quad - \varepsilon_{ijl}t_{jl}\tilde{\nabla}\varphi_i + m_{ji}\tilde{\nabla}\gamma_{ij}]\,\mathrm{d}V\,\mathrm{d}t \\
&= \int_{t_0}^{t_1}\int_V [\rho(\dot{v}_i - F_i)\tilde{\nabla}u_i + \rho(\dot{\sigma}_i - M_i)\tilde{\nabla}\varphi_i + \tilde{\nabla}W]\,\mathrm{d}V\,\mathrm{d}t \\
&= \int_{t_0}^{t_1}\int_V \rho(\dot{v}_i\tilde{\nabla}u_i + \dot{\sigma}_i\tilde{\nabla}\varphi_i)\,\mathrm{d}V\,\mathrm{d}t + \int_{t_0}^{t_1}\int_S [W - \rho(F_i u_i + M_i\varphi_i)]\mathbf{n}\,\mathrm{d}S\,\mathrm{d}t.
\end{aligned} \tag{9}$$

But

$$\mathbf{A} = \int_{t_0}^{t_1} \int_V \rho(\dot{v}_i \tilde{\nabla} u_i + \dot{\sigma}_i \tilde{\nabla} \varphi_i)\, \mathrm{d}V\, \mathrm{d}t = \int_V \rho(v_i \tilde{\nabla} u_i + \sigma_i \tilde{\nabla} \varphi_i)\, \mathrm{d}V \Big|_{t_0}^{t_1}$$

$$- \int_V \int_{t_0}^{t_1} \left\{ v_i \left[\rho \frac{\partial}{\partial t}(\tilde{\nabla} u_i) + \frac{\partial \rho}{\partial t} \tilde{\nabla} u_i \right] + \sigma_i \left[\rho \frac{\partial}{\partial t}(\tilde{\nabla} \varphi_i) + \frac{\partial \rho}{\partial t} \tilde{\nabla} \varphi_i \right] \right\} \mathrm{d}t\, \mathrm{d}V. \tag{10}$$

From the equation of continuity (1), we have

$$\frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial x_j}(\rho v_j).$$

Thus the terms $(\partial\rho/\partial t)\tilde{\nabla} u_i$ and $(\partial\rho/\partial t)\tilde{\nabla} \varphi_i$ are of higher order of smallness compared with the other terms in the two brackets, respectively, and therefore they may be omitted. Hence (10) can be written as follows:

$$\mathbf{A} = \int_V \rho(v_i \tilde{\nabla} u_i + \sigma_i \tilde{\nabla} \varphi_i)\, \mathrm{d}V \Big|_{t_0}^{t_1} - \int_{t_0}^{t_1} \int_V \rho(v_i \tilde{\nabla} v_i + j_{ij} \dot{\varphi}_j \tilde{\nabla} \dot{\varphi}_i)\, \mathrm{d}V\, \mathrm{d}t$$

$$= \int_V \rho(v_i \tilde{\nabla} u_i + \sigma_i \tilde{\nabla} \varphi_i)\, \mathrm{d}V \Big|_{t_0}^{t_1} - \int_{t_0}^{t_1} \int_V \tilde{\nabla}[\tfrac{1}{2}\rho(v_i v_i + j_{ij} \dot{\varphi}_i \varphi_j)]\, \mathrm{d}V\, \mathrm{d}t$$

$$= \int_V \rho(v_i \tilde{\nabla} u_i + \sigma_i \tilde{\nabla} \varphi_i)\, \mathrm{d}V \Big|_{t_0}^{t_1} - \int_{t_0}^{t_1} \int_S K\mathbf{n}\, \mathrm{d}S\, \mathrm{d}t. \tag{11}$$

Substituting (10) and (11) into (9), we obtain

$$\int_{t_0}^{t_1} \int_S \{[W - K - \rho(F_i u_i + M_i \varphi_i)]\mathbf{n} - (t_i \tilde{\nabla} u_i + m_i \tilde{\nabla} \varphi_i)\}\, \mathrm{d}S\, \mathrm{d}t + \int_V \rho(v_i \tilde{\nabla} u_i + \sigma_i \tilde{\nabla} \varphi_i)\, \mathrm{d}V \Big|_{t_0}^{t_1} = \mathbf{0}. \tag{12}$$

If there are two different surfaces S_1 and S_2 around some part of the crack border, then we have $S = S_1 + \Pi^+ + S_2^- + \Pi^-$, where Π^+ and Π^- mean the upper and lower crack surfaces between S_1 and S_2, and S_2^- denotes a surface obtained from S_2 by inversing its outer normals. From (12) we have

$$\int_{t_0}^{t_1} \int_S \{[W - K - \rho(F_i u_i + M_i \varphi_i)]\mathbf{n} - (t_i \tilde{\nabla} u_i + m_i \tilde{\nabla} \varphi_i)\}\, \mathrm{d}S\, \mathrm{d}t + \int_{V_0} \rho(v_i \tilde{\nabla} u_i + \sigma_i \tilde{\nabla} \varphi_i)\, \mathrm{d}V \Big|_{t_0}^{t_1} = \mathbf{0}, \tag{13}$$

here $V_0 = V_1 - V_2$ is the domain between S_1 and S_2. Since $\mathbf{n} = \mathbf{0}$, $t_i = m_i = 0$ on Π^+ and Π^-, then we obtain from (13) the following equality:

$$\int_{t_0}^{t_1} \int_{S_1} \{[W - K - \rho(F_i u_i + M_i \varphi_i)]\mathbf{n} - (t_i \tilde{\nabla} u_i + m_i \tilde{\nabla} \varphi_i)\}\, \mathrm{d}S\, \mathrm{d}t$$

$$+ \int_{V_1} \rho(v_i \tilde{\nabla} u_i + \sigma_i \tilde{\nabla} \varphi_i)\, \mathrm{d}V \Big|_{t_0}^{t_1} = \int_{t_0}^{t_1} \int_{S_2} \{[W - K - \rho(F_i u_i + M_i \varphi_i)]\mathbf{n}$$

$$- (t_i \tilde{\nabla} u_i + m_i \tilde{\nabla} \varphi_i)\}\, \mathrm{d}S\, \mathrm{d}t + \int_{V_2} \rho(v_i \tilde{\nabla} u_i + \sigma_i \tilde{\nabla} \varphi_i)\, \mathrm{d}V \Big|_{t_0}^{t_1}. \tag{14}$$

This means that eqn (12) is a path independent integral, Q.E.D.

In some cases, it is convenient to use a surface S moving with t. For example, if crack steady propagation is considered, then we have $S = S(t)$ and $V = V(t)$ and the following

Theorem 2: *The vector integral*

$$\mathbf{D}_2 = \int_{t_0}^{t_1} \int_{S(t)} \{[W + \rho(\dot{v}_i - F_i)u_i + \rho(\dot{\sigma}_i - M_i)\varphi_i]\mathbf{n} - (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i)\}\, \mathrm{d}S\, \mathrm{d}t - \int_{t_0}^{t_1} \int_{V(t)} \rho(u_i\bar{\nabla}\dot{v}_i + \varphi_i\bar{\nabla}\dot{\varphi}_i)\, \mathrm{d}V\, \mathrm{d}t, \quad (15)$$

or simply

$$\mathbf{D}_3 = \int_{S(t)} \{[W - \rho(\dot{v}_i - F_i)u_i + \rho(\dot{\sigma}_i - M_i)\varphi_i]\mathbf{n} - (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i)\}\, \mathrm{d}S - \int_{V(t)} \rho(u_i\bar{\nabla}\dot{v}_i + \varphi_i\bar{\nabla}\dot{\sigma}_i)\, \mathrm{d}V \quad (16)$$

is path independent for any $S(t)$ around some part of the crack border and any $t_1 > t_0 \geqslant 0$.

Proof: From (9) we have

$$\int_S \{[W - \rho(F_iu_i + M_i\varphi_i)]\mathbf{n} - (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i)\}\, \mathrm{d}S\, \mathrm{d}t + \int_V \rho(\dot{v}_i\bar{\nabla}u_i + \dot{\sigma}_i\bar{\nabla}\varphi_i)\, \mathrm{d}V = \mathbf{0}. \quad (17)$$

The last integral may be written as

$$\int_V \rho(\dot{v}_i\bar{\nabla}u_i + \dot{\sigma}_i\bar{\nabla}\varphi_i\, \mathrm{d}V = \int_S \rho(\dot{v}_iu_i + \dot{\sigma}_i\varphi_i)\mathbf{n}\, \mathrm{d}S - \int_V \rho(u_i\bar{\nabla}\dot{v}_i + \varphi_i\bar{\nabla}\dot{\sigma}_i)\, \mathrm{d}V. \quad (18)$$

Then substituting (18) into (17), we obtain

$$\int_S \{[W + \rho(\dot{v}_i - F_i)u_i + \rho(\dot{\sigma}_i - M_i)\varphi_i]\mathbf{n} - (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i)\}\, \mathrm{d}S - \int_V \rho(u_i\bar{\nabla}\dot{v}_i + \varphi_i\bar{\nabla}\dot{\sigma}_i)\, \mathrm{d}V = \mathbf{0}. \quad (19)$$

It is easy to prove that the integrals $\mathbf{D}_2$ and $\mathbf{D}_3$ are path independent, Q.E.D.

2.2. *Path independent integrals in micropolar elastic–plastic media*

Let W^e be the elastic deformation energy density, i.e.

$$W^e = \int (t_{ij}\, \mathrm{d}\varepsilon_{ij}^e + m_{ij}\, \mathrm{d}\gamma_{ij}^e). \quad (20)$$

In the present paper we assume that

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p, \quad (21)$$

$$\gamma_{ij} = \gamma_{ij}^e + \gamma_{ij}^p. \quad (22)$$

Now we may propose the following theorem for the crack propagation in micropolar elastic–plastic media.

Theorem 3: *The vector integral*

$$\mathbf{D}_4 = \int_{t_0}^{t_1}\int_S \{[W^e - K - \rho(F_i u_i + M_i\varphi_i)]\mathbf{n} - (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i)\}\,\mathrm{d}S\,\mathrm{d}t + \int_{t_0}^{t_1}\int_V (t_{ij}\bar{\nabla}\varepsilon_{ij}^p + m_{ij}\bar{\nabla}\gamma_{ij}^p)\,\mathrm{d}V\,\mathrm{d}t + \int_V \rho(v_i\bar{\nabla}u_i + \sigma_i\bar{\nabla}\varphi_i)\,\mathrm{d}V\Big|_{t_0}^{t_1} \quad (23)$$

is path independent for any surface S around some part of the crack border and any $t_1 > t_0 \geqslant 0$ in the elastic–plastic media.

Proof: For a closed surface not including the crack inside, by using the equality (12) we have

$$\begin{aligned}\int_{t_0}^{t_1}\int_S [-W^e\mathbf{n} + (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i)]\,\mathrm{d}S\,\mathrm{d}t &= \int_{t_0}^{t_1}\int_S [(W - W^e) - K - \rho(F_i u_i + M_i\varphi_i)]\mathbf{n}\,\mathrm{d}S\,\mathrm{d}t + \int_V \rho(v_i\bar{\nabla}u_i + \sigma_i\bar{\nabla}\varphi_i)\,\mathrm{d}V\Big|_{t_0}^{t_1} \\ &= \int_{t_0}^{t_1}\int_V (t_{ij}\bar{\nabla}\varepsilon_{ij}^p + m_{ij}\bar{\nabla}\gamma_{ij}^p)\,\mathrm{d}V\,\mathrm{d}t - \int_{t_0}^{t_1}\int_S [\rho(f_i u_i + M_i\varphi_i) - K]\mathbf{n}\,\mathrm{d}S \\ &\quad + \int_V \rho(v_i\bar{\nabla}u_i + \sigma_i\bar{\nabla}\varphi_i)\,\mathrm{d}V\Big|_{t_0}^{t_1}.\end{aligned} \quad (24)$$

Thus we get eqn (23). It is also easy to prove that (23) is path independent, Q.E.D.
From a moving surface, we may easily show the following

Theorem 4: *The vector integral*

$$\mathbf{D}_5 = \int_{t_0}^{t_1}\int_{S(t)} \{[W + \rho(\dot{v}_i - F_i)u_i + \rho(\dot{\sigma}_i - M_i)\varphi_i]\mathbf{n} - (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i)\}\,\mathrm{d}S\,\mathrm{d}t - \int_{t_0}^{t_1}\int_{V(t)} \rho(u_i\bar{\nabla}\dot{v}_i + \varphi_i\bar{\nabla}\dot{\sigma}_i)\,\mathrm{d}V\,\mathrm{d}t, \quad (25)$$

or simply

$$\mathbf{D}_6 = \int_{S(t)} \{[W + \rho(\dot{v}_i - F_i)u_i + \rho(\dot{\sigma}_i - M_i)\varphi_i]\mathbf{n} - (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i)\}\,\mathrm{d}S\,\mathrm{d}t - \int_{V(t)} \rho(u_i\bar{\nabla}\dot{v}_i + \varphi_i\bar{\nabla}\dot{\sigma}_i)\,\mathrm{d}V\,\mathrm{d}t \quad (26)$$

is path independent for any surface $S(t)$ around the crack border and any $t_1 > t_0 \geqslant 0$.

2.3. *Steady crack propagation in micropolar media*

Without loss of generality, let the propagation velocity of the field be $c\mathbf{i}$. Now we consider the surface $\bar{S}$ moving with the same velocity and use the following new coordinate system:

$$\bar{x} = x - ct, \qquad \bar{y} = y, \qquad \bar{z} = z, \qquad \bar{t} = t. \quad (27)$$

Then we have

$$v_i = \frac{\partial u_i}{\partial t} = \frac{\partial u_i}{\partial \bar{t}} - c\frac{\partial u_i}{\partial \bar{x}}, \tag{28}$$

$$\sigma_i = j_{ij}\frac{\partial \varphi_j}{\partial t} = j_{ij}\left(\frac{\partial \varphi_j}{\partial \bar{t}} - c\frac{\partial \varphi_j}{\partial \bar{x}}\right). \tag{29}$$

For the steady crack motion, we have

$$\frac{\partial}{\partial \bar{t}} = 0. \tag{30}$$

Therefore, from (29) and (28) we obtain

$$v_i = -c\frac{\partial u_i}{\partial \bar{x}}, \tag{31}$$

$$\sigma_i = -cj_{ij}\frac{\partial \varphi_i}{\partial \bar{x}}, \tag{32}$$

$$\dot{v}_i = c^2\frac{\partial^2 u_i}{\partial \bar{x}^2}, \tag{33}$$

$$\dot{\sigma}_i = c^2 j_{ij}\frac{\partial^2 \varphi_i}{\partial \bar{x}^2}. \tag{34}$$

Considering a constant path $\bar{S}$ in $\bar{x}$, $\bar{y}$, $\bar{z}$ system, from Theorem 2 and Theorem 4 we get the following path independent integrals

$$\mathbf{D}_7 = \int_{\bar{S}}\left\{\left[W + c^2\rho\left(u_i\frac{\partial^2 u_i}{\partial \bar{x}^2} + \varphi_i\frac{\partial^2 \varphi_i}{\partial \bar{x}^2}\right) - \rho(F_i u_i + M_i\varphi_i)\right]\mathbf{n} - (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i)\right\}\,\mathrm{d}\bar{S} - \int_{\bar{V}} c^2\rho\left[u_i\bar{\nabla}\left(\frac{\partial^2 u_i}{\partial \bar{x}^2}\right) + \varphi_i\bar{\nabla}\left(\frac{\partial^2 \varphi_i}{\partial \bar{x}^2}\right)\right]\mathrm{d}\bar{V} \tag{35}$$

for the micropolar elastic media and

$$\mathbf{D}_8 = \int_{\bar{S}}\left\{\left[W^e + c^2\rho\left(u_i^e\frac{\partial^2 u_i}{\partial \bar{x}^2} + \varphi_i^e\frac{\partial^2 \varphi_i}{\partial \bar{x}^2}\right) - \rho(F_i u_i^e + M_i\varphi_i^e)\right]\mathbf{n} - (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i)\right\}\,\mathrm{d}\bar{S} - \int_{\bar{V}} c^2\rho\left[u_i^e\bar{\nabla}\left(\frac{\partial^2 u_i}{\partial \bar{x}^2}\right) + \varphi_i^e\bar{\nabla}\left(\frac{\partial^2 \varphi_i}{\partial \bar{x}^2}\right)\right]\mathrm{d}\bar{V} \tag{36}$$

for the micropolar elastic–plastic media, respectively.

3. MICROPOLAR FRACTURE STATICS

For the static cases, we may from (16) and (23) obtain the following

Theorem 5: The vector integrals

$$\mathbf{S}_1 = \int_S \{[W - \rho(F_i u_i + M_i\varphi_i)]\mathbf{n} - (t_i\bar{\nabla}u_i + m_i\bar{\nabla}\varphi_i\}\ \mathrm{d}S \tag{37}$$

and

$$\mathbf{S}_2 = \int_S \{[W^c - \rho(F_i u_i + M_i \varphi_i)]\mathbf{n} - (t_i \tilde{\boldsymbol{\nabla}} u_i + m_i \tilde{\boldsymbol{\nabla}} \varphi_i)\}\,\mathrm{d}S + \int_V (t_{ij} \tilde{\boldsymbol{\nabla}} \varepsilon_{ij}^p + m_{ij} \tilde{\boldsymbol{\nabla}} \gamma_{ij}^p)\,\mathrm{d}V \quad (38)$$

are path independent for any surface S around some part of the crack border in the case of micropolar elastic media and micropolar elastic–plastic media, respectively.

Acknowledgements—This paper was read at the Symposium "Energy Release Rates and Path Independent Integral in Defect and Fracture Mechanics", which was held in January 1985 at Bad Honnef, West Germany, and organized by Prof. Dr. G. Herrmann (Stanford University) and Prof. Dr.-Ing. D. Gross (TH Darmstadt), and renewed in my visiting stay in the Institute for Mechanics of TH Darmstadt. I am deeply grateful to Prof. Dr.-Ing. D. Gross for his hearty hospitality.

REFERENCES

1. J. R. Rice, A path independent integral and the approximate analysis of strain concentration by notches and crack. *J. Appl. Mech.* **35** (1968).
2. C. Ouyang, On path independent integrals and fracture criteria in nonlinear fracture dynamics. *Appl. Math. Mech.* **3** (1982).
3. M. Z. Lu, On path independent integral in three-dimensional non-linear fracture dynamics. *Appl. Math. Mech.* **4** (1983).
4. Z. D. Chen, Energy principle of three dimensional *J*-integral. *Appl. Math. Mech.* **5** (1984).
5. D. Gross, Beiträge Zu Den Dynamischen Problemen Der Bruchmechanik (1974).
6. T. M. Dai, Conservation laws and jump conditions in linear micropolar elastodynamics, *Acta Mechanica Sinica* (in Chinese) **3** (1981).
7. F. S. Jin, The theorem of conservation of non-conservative field and the conservation laws for a certain class of continuum mechanics. *Acta Mechanica Sinica* (in Chinese) **2** (1983).
8. A. C. Eringen (Editor), *Continuum Physics*, Vol. 4. Academic Press, New York (1976).